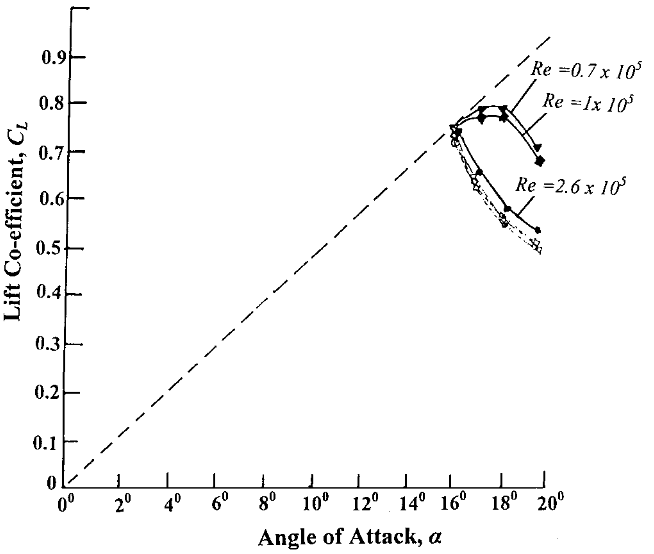

Fig. 5 Lift coefficient C_L vs angle of attack, □: acoustic excitation frequency = 2100 Hz.

c. Results at $Re = 2.6 \times 10^5$. At this Reynolds number, *Re*, the excited value for lift coefficient is found to have dropped when compared against the two earlier Reynolds numbers *Re* (Fig. 4c). The excited drag value also does not show much change. Improvement in lift and drag values were evident only at the lower angle of incidence of $\alpha = 16$ deg, where the lift and drag both improved by around 10% in the frequency range $1600 < f < 2500$ Hz, or $10 < Sr < 15$.

2. C_L vs α

To construct the C_L vs α curve, values at C_L at $\alpha = 0, 15.5, 16, 17, 18$, and 19 deg have been used (Fig. 5). For a NACA 0012 airfoil,[9] flow separation occurs at around $\alpha = 16$ deg, when the sectional or two-dimensional lift coefficient C_l drops rapidly. Noting that there is no reliable method of predicting C_L and C_D values on a wing once the flow has separated, we have made an approximate attempt to check the validity of our data just before flow separation, that is, for the case when $\alpha = 15.5$ deg using the following expression[10]:

$$C_L = 2\pi\alpha - 2C_l[(1+\delta)/AR]$$

where $0.05 \leq \delta \leq 0.25$ (Ref. 11).

For $\alpha = 15.5$ deg, $AR = 4$, and $C_l = 1.6$, with $\delta = 0.05$, the predicted $C_L \approx 0.86$; with $\delta = 0.15$, the predicted $C_L \approx 0.78$; and with $\delta = 0.25$, the predicted $C_L \approx 0.71$. At $Re \approx 0.7 \times 10^5$, the experimentally determined value for C_L at $\alpha = 15.5$ deg was found to be 0.74. Consequently, the value obtained in the experiment was considered to be of the right order for this low aspect ratio wing.

Although the C_L vs α curve (Fig. 5) for 2100-Hz acoustic excitation frequency show considerable improvement in the lift coefficient over their corresponding unexcited values, the linear relationship between the C_L and α curve is lost. The poststall drop in lift coefficient is less severe suggesting the occurrence of partial separation of flow on the wing.

IV. Conclusions

The main conclusion of this study is that acoustic excitation of boundary layer under appropriate frequencies has the potential to provide the extra energy required to modify the severe adverse pressure gradient at or near the stall. This would help the flow to remain attached to the wing and to increase the wing stall margin. In the present study, acoustic excitation on a NACA 0012 wing have shown suppression of leading-edge separation and improvement in the lift and drag coefficients over their corresponding unexcited values at $\alpha = 16, 17$, and 18 deg, that is, 3 deg beyond stall angle of the unexcited wing. This study also shows some dependence of the beneficial acoustic frequencies on Reynolds number, with higher frequencies required for higher Reynolds number. Our study, however, did not find significant improvements at $Re = 2.6 \times 10^5$, possibly because the maximum equivalent Strouhal number during the test was low. To confirm the presence of stall suppression at $Re = 2.6 \times 10^5$ at a *Sr* of around 40 as displayed for the other two Reynolds numbers would require an excitation frequency in excess of 6000 Hz, which was not available during this study.

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Lift and Drag Characteristics of a Supersonic Biplane Configuration

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Introduction

THE requirements of man's initial powered flight endeavours were ably met by the biplane configuration. However, subsequent structural and aerodynamic advances found the biplane falling into disfavor in the early 1930s. For a fixed wing span biplanes do possess aerodynamic efficiency advantages as compared to a monoplane. At a given lift coefficient and assuming elliptic loading, the vortex drag of a biplane tends to half that of a monoplane as the separation distance between the wings tends to infinity. The biplane captures a larger volume of air that is accelerated down to generate the lift impulse, so reducing the downwash velocity and hence the kinetic energy imbued to the accelerated fluid.

Biplanes have several interesting characteristics that are summarized below. Prandtl and Tietjens[1] has shown for unstaggered biplanes (i.e., neither wing extends in front of the other) the drag increments caused by the mutual influence of the wings are equal and are always additive. For positive stagger (the upper wing in front of the lower wing) the upper wing increases the downwash on the lower wing so increasing its drag; vice versa for the effect of the lower wing on the upper wing. Munk[2] has shown that the total mutual induced drag of a biplane for a fixed gap is independent of the amount of stagger (Munk's stagger wing theorem). This theorem is only valid if the two wing's lift distributions are

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unaltered (through varying the wing's α). For multiplanes minimum drag of the system is achieved when the induced downwash velocities on the wings are equal and constant along the span, as is the case for a monoplane. Drag is also minimized by matching the span of the biplane's wings. The forthcoming discussion will address the potential of coupling biplane aerodynamics with those of delta wings to yield an advanced supersonic planform. Busemann proposed a nonlifting supersonic biplane type configuration that incurred no wave drag from thickness. The biplane functioned through internal wave cancellation eliminating wave-thickness drag; complete cancellation occurring at only one Mach number and wave angle.

Discussion

In supersonic flight the drag of a configuration is generally composed of skin friction, wave drag due to thickness as well as wave drag due to lift and vortex drag. Wave drag can be reduced significantly by selecting the wing sweep and cruise Mach number such that the wing's leading edge remains subsonic (the freestream velocity component normal to the leading edge is subsonic). In this case leading-edge suction is retained and reduces drag.

In supersonic flight the vortex drag of a wing generally follows the same principles as in subsonic flight, i.e., for minimum drag the downwash trace should be constant over the wing in the characteristic envelope. To this end, Jones[3] has shown that an oblique elliptic planform wing has the theoretical minimum lift-dependent drag for a planar supersonic configuration, although Jones initially proposed this configuration as a concept to attenuate sonic boom overpressures. Tests of this configuration have unfortunately revealed serious aeroelastic and control problems.

Munk[2] and Cone[4] have shown that in subsonic flow numerous nonplanar wing forms (e.g., biplanes, wings with winglets, etc.) possess greater efficiency than the optimal planar wing for a fixed wing span. However, for biplanes in a practical configuration the proximity of the wings to each other causes interference that reduces the efficiency of the system. For a suitably configured vehicle in supersonic flight, it is feasible that the wings could effectively operate independently even though they may be in fairly close proximity. This would be achieved by nonintersecting Mach cones in the vicinity of the wing as shown in Fig. 1. In subsonic flow the interference effects between the wings reduce as the wing gap or separation increases. As an example, consider a biplane consisting of two 75-deg sweep delta wings.[5] Interference effects are only reduced to appreciable levels for wing gaps z over wing root chords (c_r), ($z/c_r > 1$), in subsonic flow (see Fig. 2). In supersonic flow, at a Mach number of 3, for $z/c_r > 0.7$ the individual wing's Mach cones do not intersect. Similarly, at $M = 4$, $z/c_r > 0.52$. At these conditions then, each wing is unaware of the presence of the other wing, and they operate independently without any mutual interference. It is instructive to estimate the potential reductions in drag that can be achieved by the supersonic biplane configuration. Geometric considerations suggest that for the following analysis to be valid it would be necessary for the two delta wings to be joined at their centerlines.

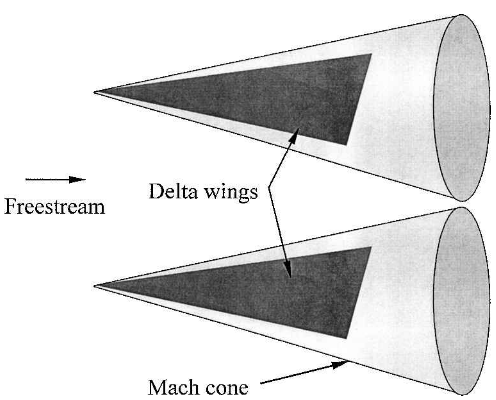

Fig. 1 Proposed supersonic biplane configuration at cruise conditions.

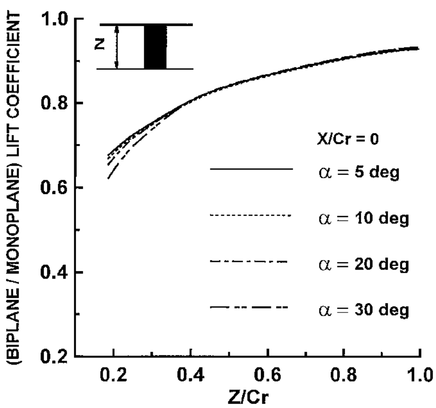

Fig. 2 Effect of biplane gap z on interference, subsonic flight conditions.[5]

Following Polhamus,[6] the lift curve slope for a slender delta wing in supersonic flight can be estimated as

$$C_{L\alpha} = \pi AR/2E(k) \tag{1}$$

where AR is the wing aspect ratio and the elliptic integral is given by

$$E(k) = \int_0^{\pi/2} [1 - k^2 \sin(z)^2]^{\frac{1}{2}} \, \mathrm{d}z$$

with $k = [1 - \beta/\tan(\Lambda)]^{1/2}$ and $\beta = (M^2 - 1)^{1/2}$, where M is Mach number and Λ is the wing leading-edge sweep angle. Polhamus has shown that the vortex lift for a supersonic delta with leading-edge separation can be estimated using

$$k_v = \frac{\pi\{[16 - (AR\beta)^2](AR^2 + 16)\}^{\frac{1}{2}}}{16E(k)^2} \tag{2}$$

Assume that the biplane deltas are operating at cruise condition such that their Mach cones are not intersecting (see Fig. 1), and the flow is attached at the wing's leading edges. At low angles of attack (typical of cruise operating conditions), the lift coefficient C_L is given by

$$C_L = C_{L\alpha}\alpha \tag{3}$$

Attached Flow

The lift-dependent drag coefficient C_{DL} (assuming attached flow at the wing leading edge) then follows from Polhamus' suction analogy as

$$C_{\mathrm{DL}} = [C_{L\alpha} - k_v \cos(\Lambda)]\alpha^2 \tag{4a}$$

or

$$C_{\mathrm{DL}} = \left[\frac{1}{C_{L\alpha}} - \frac{k_v \cos(\Lambda)}{C_{L\alpha}^2}\right] C_L^2 \tag{4b}$$

Equations (4a) and (4b) are equivalent to the following expression derived by Brown[7] for the lift-dependent drag corresponding to attached flow over a delta wing:

$$C_{\mathrm{DL}} = \left(C_L^2/\pi AR\right)\left[2(1 + \lambda/\pi) - \sqrt{1 - \beta^2 \tan(90^\circ - \Lambda)^2}\right] \tag{5}$$

where λ is a geometric parameter dependent on $\tan(90^\circ - \Lambda)/\tan(\mu)$. For the limiting case as the wing becomes infinitely slender,

$$\underset{\Lambda \to 90^\circ}{C_{\mathrm{DL}}} = C_L^2/2C_{L\alpha} \tag{6}$$

which is the result given by Jones,[3,8] indicating the resultant inviscid force on the wing is inclined at $\alpha/2$ and the wing has elliptic loading. Ultimately, as the Mach number increases, the wing leading edge becomes supersonic indicating the Mach cone coincides with leading edge. For a planar wing this implies that the resultant inviscid force is perpendicular to the wing, such that the lift-dependent drag is given by

$$C_{\mathrm{DL}} = C_L^2 / C_{L\alpha} \tag{7}$$

Consider two geometrically similar delta wings such that their Mach cones do not intersect in the vicinity of the wing. As each wing is operating unaware of the presence of the other wing, each wing will possess the same lift curve slope $C_{L\alpha}$ of the equivalent monoplane. Assuming that the biplane and monoplane are required to generate the same lift and each possess the same lift curve slope, each wing of the biplane cellule is then required to generate half of the lift of the monoplane. It follows that

$$C_{\mathrm{Lbip}} = C_{\mathrm{Lmono}} = C_{L\alpha}\alpha_{\mathrm{mono}} = 2(C_{L\alpha}\alpha_{\mathrm{bip}}) \tag{8}$$

and $\alpha_{\mathrm{bip}} = \alpha_{\mathrm{mono}}/2$ for the same lift. Thus

$$C_{\mathrm{DLmono}} = [C_{L\alpha} - k_v \cos(\Lambda)]\alpha_{\mathrm{mono}}^2 \tag{9}$$

and

$$C_{\mathrm{DLbip}} = 2[C_{L\alpha} - k_v \cos(\Lambda)]\alpha_{\mathrm{bip}}^2 \tag{10}$$

where the 2 preceding the second expression follows from the cellule consisting of two wings. Substituting for α_{bip} and dividing the two equations yields

$$C_{\mathrm{DLbip}} = \tfrac{1}{2} C_{\mathrm{DLmono}} \tag{11}$$

Thus, for equal size wings the lift-dependent drag of the biplane cellule will be half that of the monoplane provided the Mach cones from the two wings do not intersect and the flow does not separate. The drag reduction is caused by the biplane wings influencing or entraining a larger volume of fluid, thereby requiring lower wake velocities (and hence kinetic energy) to accelerate the air to generate the lift impulse. Alternatively, the biplanes are at half the angle of attack of the monoplane, thus reducing the rearward component of the lift and hence its contribution to drag. It is unlikely in operation that the flow would remain attached at the wing's leading edge, as the extreme sweep of a thin delta wing generally causes flow separation at moderate angles of attack.

Separated Flow

If the flow separates, then vortex formation can occur.[6] In this instance both lift and drag will increase. Of significance though is the effect of biplane aerodynamics on the associated vortex drag. At low angles of attack, the lift coefficient, including vortex lift effects, is given by

$$C_L = C_{L\alpha}\alpha + k_v\alpha^2 \tag{12}$$

The inviscid lift-dependent drag coefficient is given by (assuming zero leading-edge suction)

$$C_D = C_L\alpha \tag{13}$$

Combining the preceding equations yields

$$C_D = C_{L\alpha}\alpha^2 + k_v\alpha^3 \tag{14}$$

The first term on the right is the drag due to the attached flow lift coefficient assuming zero leading-edge suction, which at low α is equivalent to the attached flow lift with leading-edge suction examined earlier. The second term is the drag associated with the vortex lift. For a noninterfering biplane where each wing has equal area to the comparative monoplane, $C_{\mathrm{Lbip}/2} = C_{\mathrm{Lmono}}/2$ for each wing. Consequently, to evaluate the biplane drag it is necessary to determine the corresponding α at which each biplane wing develops $C_{\mathrm{Lmono}}/2$.

It follows that

$$C_{\mathrm{Lmono}}/2 = C_{L\alpha}\alpha_{\mathrm{bip}} + k_v\alpha_{\mathrm{bip}}^2$$

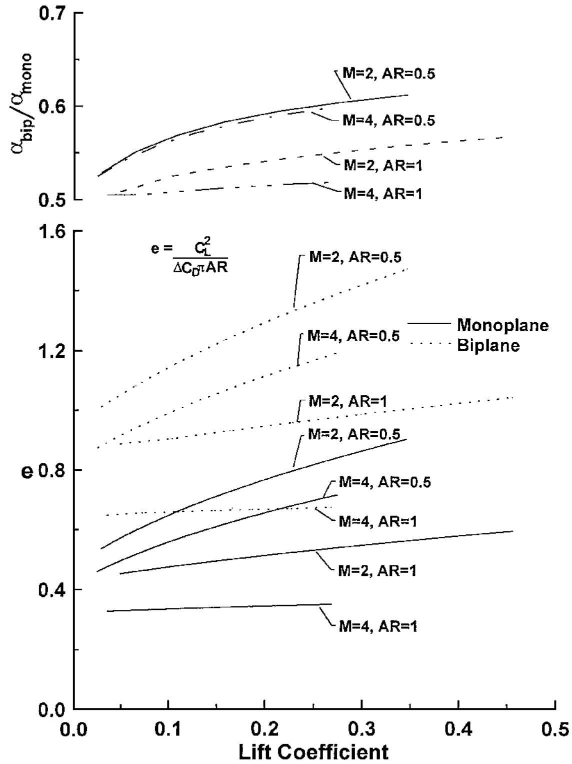

Fig. 3 Efficiency and required angle of attack of noninterfering biplane and monoplane generating equivalent lift.

Solving for α_{bip} gives

$$\alpha_{\mathrm{bip}} = \left[-C_{L\alpha} + \sqrt{C_{L\alpha}^2 + 2k_v\left(C_{L\alpha}\alpha_{\mathrm{mono}} + k_v\alpha_{\mathrm{mono}}^2\right)}\right]\Big/ 2k_v \tag{15}$$

Typical results from Eq. (15) are presented in the upper plot of Fig. 3. The incidence of the biplane for the same total lift (as a monoplane of equal area to one of the biplane wings) is approximately 53% of the monoplane and increases with lift. The low $C_{L\alpha}$ associated with the very slender $AR = 0.5$ wings result in a corresponding biplane requiring greater incidence relative to the monoplane then the less slender $AR = 1$ wings. To evaluate the effect of the reduced incidence required by the biplane on the drag, a representative value of $\alpha_{\mathrm{bip}}/\alpha_{\mathrm{mono}} = 0.53$ can be substituted into Eq. (14). This yields $C_{\mathrm{Dbip}} = 2(C_{L\alpha}0.28\alpha_{\mathrm{mono}}^2 + k_v 0.15\alpha_{\mathrm{mono}}^3)$ for both biplane wings. Thus for separated flow with vortex lift, the biplane generates approximately 56% of the attached flow lift-dependent drag and only 30% of the drag associated with vortex lift. The skin-friction drag would be approximately twice that of the monoplane (twice the wetted area).

Figure 3 presents the wing efficiency e evaluated using $e_{\mathrm{bip}} = C_L/\pi AR\alpha_{\mathrm{bip}}$ and $e_{\mathrm{mono}} = C_L/\pi AR\alpha_{\mathrm{mono}}$. Cases are presented for noninterfering deltas with $AR = 0.5$ ($\Lambda = 82.9$ deg) and $AR = 1$ ($\Lambda = 76$ deg) at Mach 2 and 4. The data clearly show that the biplane yields a significant improvement in efficiency over the monoplane. Efficiency e of the more slender wing $AR = 0.5$ shows greater sensitivity to lift coefficient than $AR = 1$. This effect is due to the larger proportion of vortex lift (and its α^2 dependence) to total lift as slenderness increases. It can be seen from Fig. 3 that increasing the Mach number reduces the wing's efficiency, as a result of a reduction in $C_{L\alpha}$ and k_v. k_v tends to zero as the wing's leading edge becomes supersonic as a result of the elimination of upwash. Increasing C_L increases e due to increased lift from the vortex sheets. For separated flow, if no vortex lift is developed the wing efficiency reduces to that for attached flow, e.g., $\alpha_{\mathrm{bip}} = \alpha_{\mathrm{mono}}/2$ and $e_{\mathrm{bip}} = 2e_{\mathrm{mono}}$, although the actual magnitude of e_{bip} and e_{mono} reduces without vortex lift.

For a biplane cellule where each wing possesses half of the area of the monoplane wing, the biplane would generate the same lift and possess the same lift-dependent drag as the monoplane at the same angle of attack (provided the Mach cones do not intersect). However, for delta wings the root chord of the biplane wings would now measure 0.71 of the monoplanes. Thus, for a supersonic biplane configuration where the Mach cones do not intersect the drag of the biplane will equal that of the monoplane; however, the biplane's chord will be 70% of the monoplanes. The second example is interesting in that by reducing the wing chord this configuration should naturally benefit viscous drag-reduction efforts by reducing the wing's operational Reynolds number. Thus, increases in achievable extents of laminar flow (at high-altitude cruising conditions) on the wing may not require any active control systems. The use of delta wings with a root chord between $0.7\ c_{\text{rmono}} < c_{\text{rbip}} < c_{\text{rmono}}$ will yield advantages in terms of both lift-dependent and possibly skin-friction drag and would present a useful compromise.

As already mentioned, Jones[3] has shown that an oblique elliptic wing is the most efficient supersonic aerodynamic planform. He has shown that the drag coefficient of this type of wing is roughly half that of a delta (with a subsonic leading edge) generating the same lift. However, as just shown, noninterfering delta planform biplanes can possess half the drag of a monoplane delta. Consequently, under optimal cruise conditions a biplane delta wing and an oblique elliptic wing should have comparable aerodynamic performance. Additionally, the delta wings would have the generally benign handling characteristics (at low to moderate α) associated with these wings, whereas the oblique wing is plagued by various operational drawbacks.

As the biplane configuration accelerates up to its cruise conditions, it would pass through a Mach-number range where the Mach cones from the wings would interact. This could potentially cause drag increases and may limit the ability of the aircraft to reach operating conditions. The supersonic biplane configuration requires study to determine these interactions.

Conclusions

A theoretical analysis has been undertaken to determine the lift and vortex drag of delta wings in a biplane configuration. It is assumed that noninterfering Mach cones in the vicinity of the wings results in the wings effectively operating without mutual interference. For biplane cellules meeting this criteria, it is shown that the vortex drag of the biplane configuration is half that of a monoplane wing generating the same lift coefficient for the attached flow drag component and is approximately 30% of that of the monoplane for the drag associated with the presence of leading-edge vortices.

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